II Semester B.A./B.Sc. Examination, May/June 2016
(CBCS) (Fresh + Repeaters)
(2014 –15 & Onwards)
MATHEMATICS (Paper – II)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

a) The binary operation $*$ on the set of positive rational numbers Q^+ is defined by $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. Find the identity element of the set Q^+ and the inverse of 3.

b) In a group $(G, *)$, $\forall a, x, y \in G$. Prove that $x * a = y * a \Rightarrow x = y$.

c) Find the angle between the radius vector and the tangent to the curve $r = ae^{\theta \cot \alpha}$.

d) Find the length of the polar subnormal at the point $\theta = \pi/6$, for the curve $r = a\cos 2\theta$.

e) Find the derivative of the arc-length $\frac{ds}{d\theta}$ for the curve $r = a\theta$.

f) Find the area bounded by the x - axis and the curve $y = c\cosh\left(x/c\right)$ between $x = 0$ and $x = a$.

g) Find the integrating factor of : $\frac{dy}{dx} + \sec x . y = \tan x$.

h) Verify for exactness : $\left(x + \frac{y^3}{x^2}\right)dx - \frac{3y^2}{x}dy = 0$.

P.T.O.

PART – B

Answer **any one full** question. **(1×15=15)**

2. a) Let (G, *) be a group and $a, b \in G$, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

   b) Show that $(z_6, \oplus_6)$ where $z_6 = \{0, 1, 2, 3, 4, 5\}$ is a group.

   c) Show that the set C of all complex numbers is a group under addition of complex numbers.

OR

3. a) Prove that a non-empty subset H of a group (G, *) is a subgroup of G if and only if $\forall a, b \in H,\ a * b^{-1} \in H$.

   b) Show that the set of square roots of unity is a subgroup of the group of fourth roots of unity under multiplication of complex numbers.

   c) In a set A = {1, 2, 3} let $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ find $f \circ g$, $g \circ f$ and $(f \circ g)^{-1}$.

PART – C

Answer **any two full** questions. **(2×15=30)**

4. a) With usual notations, prove that $\tan\phi = r\frac{d\theta}{dr}$, for the polar curve, $r = f(\theta)$.

   b) Find the co-ordinates of the centre of curvature at (x, y) for the curve $y = a\cosh\left(x/a\right)$.

   c) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where a & b are connected by the relation $ab = c^2$.

OR

5. a) Prove that with usual notations, the radius of curvature of the curve y = f(x) is $\rho = \frac{\left(1 + y_1^2\right)^{3/2}}{y_2}$.

b) Show that the following curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect orthogonally.

c) Find the Pedal equation of the curve : $r^n = a^n \sin n\theta$.

6. a) Find all the asymptotes of the curve : $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$.

b) Find the surface area generated by the revolution of an arc of the catenary $y = c\cosh\left(\frac{x}{c}\right)$ about the x – axis between x = 0 and x = a.

c) Find the position and nature of the double points of the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$.

OR

7. a) Find the perimeter of the cardioide $r = a(1+\cos\theta)$.

b) Find the position and nature of the double points on the curve $x^3 + y^3 = 3axy$.

c) Find the volume of the solid obtained by revolving one arch of the cycloid : $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ about the x – axis.

PART – D

Answer **any one full** question : **(1×15=15)**

8. a) Solve the differential equation : $(1+x^2)dy + (y - \tan^{-1}x)dx = 0$.

b) Solve : $y = 2px + y^2 p^3$.

c) Verify for exactness and solve :

$(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$.

OR

9. a) Solve : $\frac{dy}{dx} - \frac{2}{x}.y = \frac{1}{x^3}.y^2$.

b) Find the general and singular solution of : $\sin px \cos y - \cos px \sin y = p$.

c) Show that the family of curves $x^2 = 4a(y + a)$ is self orthogonal.